Published by Institute of Physics Publishing for SISSA

Received: *May 22, 2006*
Revised: *June 12, 2006*
Accepted: *June 21, 2006*
Published: *July 17, 2006*

Towards a realistic type IIA $T^6/\mathbb{Z}_4$ orientifold model with background fluxes, part 1. Moduli stabilization

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Abstract: We apply the methods of DeWolfe et al. (hep-th/0505160) to a $T^6/\mathbb{Z}_4$ orientifold model. This is the first step in an attempt to build a phenomenologically interesting meta-stable de Sitter model with small cosmological constant and standard model gauge groups.

Keywords: Flux compactifications, Superstring Vacua.

JHEP07(2006)027

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1. Introduction

There have been some longstanding, unsolved problems when it comes to realistic model building within the framework of Calabi-Yau (CY) compactifications of superstring theories, namely

- supersymmetry breaking,
- the moduli problem,
- a small, but nonvanishing cosmological constant $\Lambda > 0$ [1−3], indicating an asymptotic de Sitter (dS) type universe. Moreover, $w < -\frac{1}{3}$ indicates an accelerated expansion.

Especially the last point seems to pose a serious challenge for string theory, because (eternal) de Sitter type universes, due to the existence of event horizons, are believed to necessitate a finite number of physical degrees of freedom (resulting in finite dimensional Hilbert spaces) [4, 5], which appears impossible to reconcile with string theory. Moreover, compactifications of string theory on Calabi-Yau 3-folds to four spacetime dimensions generically produce a large number of massless moduli (scalar fields) which we do not observe in nature. However, a recent proposal by Kachru, Kallosh, Linde and Trivedi (KKLT) [6] manages to address all of the above stated difficulties at once. The authors outline a way to produce a nontrivial (scalar) potential for *all* CY moduli, resulting in supersymmetric anti-de Sitter (AdS) vacua in which all moduli are stabilized. To achieve this, the authors start with

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a warped compactification of a type IIB orientifold with background fluxes as discussed in [7]. There it was shown that by turning on appropriate R-R and NS-NS 3-form fluxes $\hat{F}_{(3)}$ and $\hat{H}_{(3)}$, it is possible to fix both the complex structure moduli z^α and the axiodilaton $\tau := C_{(0)} + \mathrm{i}e^{-\hat{\phi}}$. However, owing to the fact that the flux-induced superpotential[1] $W_0^{\mathrm{IIB}} = \int_{CY_3} \hat{G}_{(3)} \wedge \Omega$ [8] does not depend on the Kähler moduli of the compactification manifold, one is forced to include nonperturbative corrections to W in order to generate a potential for those moduli. KKLT argue that this can be achieved generically in their class of models by one of two effects: Euclidean $D3$-brane instantons wrapping divisors of arithmetic genus equal to one [9] or gaugino condensation in the gauge theory living on a stack of coinciding $D7$-branes wrapping 4-cycles of the internal CY [10, 11]. Both effects can be shown to lead to stabilization of the remaining Kähler moduli. As a matter of fact, the condition on the arithmetic genus of the divisors can be relaxed in the presence of fluxes, as was discovered recently by several authors (see e.g. [12]). In the final step of the KKLT construction it is argued that by adding $\overline{D3}$-branes to the setup in a suitable fashion, it is possible to break supersymmetry in such a way that the vacuum is lifted to a dS vacuum with a discretely tunable cosmological constant[2]. It is, however, important to note that the dS vacua in question are only local minima of the $\mathcal{N} = 1$ supergravity scalar potential for the relevant moduli. There always exists a global minimum, the Dine-Seiberg runaway vacuum in the large volume or decompactification limit. Therefore the dS vacua are only metastable, albeit at cosmological time scales, thus evading the above mentioned problems concerning eternal de Sitter spacetimes.

The program outlined by KKLT triggered a myriad of work within the framework of type IIB orientifold compactifications [14 − 17]. Several important refinements to the original KKLT proposal were made, e.g., V. Balasubramanian, F. Quevedo and collaborators [18 − 20] realized that it is inconsistent (at least generically) to neglect the perturbative α'-corrections to the Kähler potential. Stated differently, by including these corrections, one can prove the existence of AdS vacua (even nonsupersymmetric ones) and the validity of the construction for a much broader range of parameters as compared to the original proposal without perturbative corrections.

In recent months, several authors have studied various aspects of the KKLT program in the framework of type IIA orientifold compactifications [21 − 25]. One important difference compared to the type IIB case is that here, as we shall see below, the flux-induced superpotential W_0^{IIA} contains contributions both from the complex structure as well as the Kähler moduli. Therefore, one Kähler modulus of the untwisted sector gets projected out by the orientifold [3] without having to consider nonperturbative instanton corrections. Another worthwhile observation is that whereas in the type IIB scenario the fluxes are highly constrained by the tadpole cancelation condition for the $\hat{C}_{(4)}$-field, this is not true in the IIA setup, where some of the fluxes, namely $\hat{F}_{(2)}$ and $\hat{F}_{(4)}$, are left unaffected and thus unconstrained by the $\hat{C}_{(7)}$-tadpole cancelation condition [21, 22, 24].

[1]Here we introduce the complexified 3-flux $\hat{G}_{(3)} := \hat{F}_{(3)} - \tau \hat{H}_{(3)}$.

[2]This tuning can be achieved by turning on appropriate fluxes through cycles in the internal manifold.

[3]One can stabilize all the complex structure moduli but only one linear combination of the axions.

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In the present paper we work out and discuss in some detail the moduli stabilization for a specific $T^6/\mathbb{Z}_4$ orientifold model. It has the prospect of yielding a viable stringy realization of the ingredients needed for a realistic description of particle physics, namely the correct particle spectrum (SM or MSSM) combined with desired cosmological features ($\Lambda > 0$). These more advanced issues will be addressed in future research. In this paper we find supersymmetric and nonsupersymmetric AdS vacua in which all moduli are stabilized. Moreover we exhibit some vacua in which one Kähler modulus remains unfixed (flat direction), although we have turned on generic fluxes.

The paper is organized as follows: We begin by introducing the basic setup and the construction of the orientifold model in section 2. Section 3 contains a detailed discussion of moduli stabilization via flux-induced potentials for the moduli of the untwisted sector. We present two different approaches to this problem: First, starting from ten-dimensional massive type IIA supergravity, we obtain the four-dimensional effective scalar potential by Kaluza-Klein reduction. Second, we solve supersymmetric F-flatness conditions in the language of four-dimensional $\mathcal{N}=1$ supergravity, yielding supersymmetric AdS vacua. We extend our considerations to the twisted sector moduli fields in section 4, followed by some conclusions and an outlook in section 5.

2. Basic setup

2.1 The $T^6/\mathbb{Z}_4$ orientifold

In this section, we outline the properties of the type IIA orientifold model under investigation, namely an orientifolded $T^6/\mathbb{Z}_4$ orbifold that preserves $\mathcal{N}=1$ supersymmetry. A detailled discussion of this model can be found in [28].

The $T^6/\mathbb{Z}_4$ orbifold. As a first step, we want to compactify type IIA string theory on an $T^6/\mathbb{Z}_4$ orbifold background[4]. Let us start by describing the orbifold construction, following [28, 29]. It is important to use a lattice for the T^6 that implements a crystallographic action of the cyclic group. Therefore one chooses the root lattice of an appropriate Lie algebra. In the $\mathbb{Z}_4$ case under investigation the appropriate choice is $\mathrm{SU}(2)^6$. Unlike the more complicated orbifolds with quotient group $\mathbb{Z}_N$ for $N > 6$ [31], in the case of $\mathbb{Z}_4$, the root lattice of the Lie algebra allows a choice of complex structure in such a way that the torus factorizes as $T^6 = T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)}$. We parameterize it by three complex coordinates $z^i,\ i \in \{1,2,3\}$, together with the periodic identifications

$$z^i \sim z^i + \pi_{2i-1} \sim z^i + \pi_{2i},\ i \in \{1,2,3\}, \tag{2.1}$$

where the π_k denote the fundamental 1-cycles of the three 2-tori. The $\mathbb{Z}_4$ action on the torus T^6 is given by

$$\Theta : (z^1, z^2, z^3) \mapsto (\alpha z^1, \alpha z^2, \alpha^{-2} z^3), \tag{2.2}$$

[4]The $T^6/\mathbb{Z}_4$ orbifold is among those studied in [26, 27] and has been shown to admit consistent string propagation, e.g., preserving modular invariance.

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sector:	untwisted	Θ, Θ^3-twisted	Θ^2-twisted	$\sum$
fixed points/type:	—	16 $\mathbb{Z}_4$	12 $\mathbb{Z}_2$+ 4 $\mathbb{Z}_4$ ($\mathbb{Z}_2$)	—
complex structure:	1	—	6+0	1+6
Kähler:	5	16	6+4	5+26

Table 1: List of complex structure and Kähler moduli.

where $\alpha = e^{\mathrm{i}\pi/2} = \mathrm{i}$ is a fourth root of unity and $\Theta^4 = \mathbb{1}$. This action preserves $\mathcal{N} = 2$ supersymmetry in four dimensions, implying that the orbifold is actually a singular limit of a Calabi-Yau 3-fold. The Hodge numbers are given by $h^{1,1} = 31$ and $h^{2,1} = 7$, yielding the number of Kähler and complex structure moduli before the orientifold projection. Table 1 lists how the complex structure and Kähler moduli appear in the different sectors of the orbifold.

The Euler characteristic turns out to be

$$\chi(T^6/\mathbb{Z}_4) = 2(h^{1,1} - h^{2,1}) = \frac{1}{|\mathbb{Z}_4|} \sum_{gh=hg} \chi(g,h) = 48, \tag{2.3}$$

where $\chi(g,h)$ denotes the Euler characteristic of the subspace invariant under both g and h. $|\mathbb{Z}_4| = 4$ is the order of the group. The sum runs over all pairs of elements of the Abelian subgroup of the quotient group; here, since $\mathbb{Z}_4$ is Abelian, the sum runs over the sixteen pairings involving all four group elements[5].

The orientifold model. As in [28, 29], we construct a $T^6/\mathbb{Z}_4$ orientifold by modding out by $\mathcal{O} = \Omega_p(-1)^{F_L}\sigma$, where Ω_p denotes worldsheet parity and $(-1)^{F_L}$ stands for left-moving fermion number. There are two distinct choices for the antiholomorphic[6] involution σ on each of the T^2. We choose[7]

$$\begin{aligned} \sigma &: z^1 \mapsto \bar{z}^1, \\ \sigma &: z^2 \mapsto \alpha\bar{z}^2, \\ \sigma &: z^3 \mapsto \bar{z}^3. \end{aligned} \tag{2.4}$$

For the first two tori, the complex structure is fixed to be i, so $z^i = x^i + \mathrm{i}y^i$, $i = 1, 2$. On the third torus the $\mathbb{Z}_4$ action does not fix the complex structure $z^3 = x^3 + \mathrm{i}U_2 y^3$. The tori and our choices of fundamental 1-cycles are shown in figure 1. After the orientifold projection we have an $O6$ orientifold plane wrapping the invariant special Lagrangian 3-cycles in

[5]The actions of $\Theta^1, \Theta^2, \Theta^3$ all yield 16 fixed points. However, four pairs of elements, namely those involving combinations of $\Theta^0 = \mathbb{1}$ and $\Theta^2 : (z^1, z^2, z^3) \mapsto (\alpha^2 z^1, \alpha^2 z^2, z^3)$, leave at least one of the T^2 factors invariant, thus not contributing to the sum, as $\chi(T^6) = \chi(T^2) = 0$.

[6]In type IIA superstring theory, the involutive symmetry σ has to be chosen to be antiholomorphic, since the left-moving space-time supercharge corresponds to the holomorphic 3-form, whereas the right-moving space-time supercharge corresponds to the antiholomorphic 3-form. In the type IIB case both supercharges are related to the holomorphic 3-form, thus necessitating a holomorphic involution. [33]

[7]This is the **ABB** model discussed in detail in [29].

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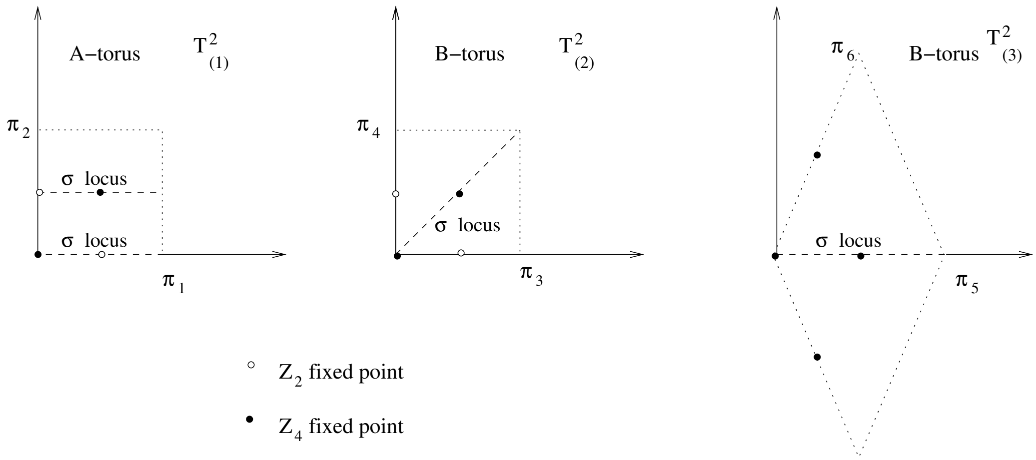

Figure 1: Tori of the **ABB** model.

projection	fixed point set
$\mathcal{O}$	$2(\pi_{135} + \pi_{145})$
$\mathcal{O}\Theta$	$2\pi_{145} + 2\pi_{245} - 4\pi_{146} - 4\pi_{246}$
$\mathcal{O}\Theta^2$	$2(\pi_{235} - \pi_{245})$
$\mathcal{O}\Theta^3$	$-2\pi_{135} + 2\pi_{235} + 4\pi_{136} - 4\pi_{236}$

Table 2: Invariant cycles in each sector of the **ABB** model.

$T^6/\mathbb{Z}_4$ and filling the four noncompact dimensions. For reference, we have summarized the invariant cycles in each sector in table 2. There we employ the notation

$$\pi_{ijk} := \pi_i \otimes \pi_j \otimes \pi_k, \tag{2.5}$$

where π_{2i-1} and π_{2i} denote the two fundamental 1-cycles of the three 2-tori $T_i^2,\ i \in \{1,2,3\}$ (see figure 1). The $\mathbb{Z}_4$ action maps the cycles invariant under $\mathcal{O}$ and $\mathcal{O}\Theta^2$ into each other and likewise for the other two cycles. Therefore there are two invariant 3-cycles that are both wrapped once by the $O6$-plane:

$$[a_0] := 2(\pi_{135} + \pi_{145} + \pi_{235} - \pi_{245}) \tag{2.6}$$

$$[a_1] := 4(\pi_{136} - \pi_{146} - \pi_{246} - \pi_{236}) + 2(-\pi_{135} + \pi_{145} + \pi_{245} + \pi_{235}) \tag{2.7}$$

In addition, there will be exceptional 3-cycles related to the blow-ups of the fixed point singularities (cf. section 4).

The $O6$-plane contributes to a $\hat{C}_{(7)}$-tadpole that has to be canceled either by introducing $D6$-branes or by turning on appropriate fluxes. This issue will be addressed in the next section. It is important to note that both the $O6$-plane and the $D6$-branes can be chosen to preserve/break the same supersymmetry. Thus, we are left with $\mathcal{N} = 1$ supersymmetry in four dimensions.

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2.2 Moduli and fluxes

Before embarking on the task of generating appropriate potentials by turning on fluxes, let us collect the relevant moduli fields, forms and cycles appearing in our construction. We start out by taking a closer look at the 3-cycles in the game. Since $b^3_{\text{untw.}} = 2+2h^{2,1}_{\text{untw.}} = 4$, we expect four 3-cycles from the untwisted sector. This fits nicely with the observation that the only $(2,1)$-form invariant under the $\mathbb{Z}_4$-action is $dz^1 \wedge dz^2 \wedge d\overline{z}^3$, so that the four 3-cycles are simply the duals of the holomorphic $(3,0)$-form Ω, the antiholomorphic $(0,3)$-form $\overline{\Omega}$, the $\mathbb{Z}_4$-invariant $(2,1)$-form and the associated $\mathbb{Z}_4$-invariant $(1,2)$-form.
The 1-cycles yield the following behavior under the $\mathbb{Z}_4$-action,

$$\begin{aligned}
\Theta^1: \qquad & \pi_1 \mapsto +\pi_2, \pi_3 \mapsto +\pi_4, \pi_5 \mapsto -\pi_5, \\
& \pi_2 \mapsto -\pi_1, \pi_4 \mapsto -\pi_3, \pi_6 \mapsto -\pi_6, \\
\Theta^2: \qquad & \pi_1 \mapsto -\pi_1, \pi_3 \mapsto -\pi_3, \pi_5 \mapsto +\pi_5, \\
& \pi_2 \mapsto -\pi_2, \pi_4 \mapsto -\pi_4, \pi_6 \mapsto +\pi_6, \\
\Theta^3: \qquad & \pi_1 \mapsto -\pi_2, \pi_3 \mapsto -\pi_4, \pi_5 \mapsto -\pi_5, \\
& \pi_2 \mapsto +\pi_1, \pi_4 \mapsto +\pi_3, \pi_6 \mapsto -\pi_6,
\end{aligned} \tag{2.8}$$

leading to the following $\mathbb{Z}_4$-invariant combination of 3-cycles

$$\begin{aligned}
\rho_1 &:= 2(\pi_{135} - \pi_{245}), \qquad & \widetilde{\rho}_1 &:= 2(\pi_{136} - \pi_{246}), \\
\rho_2 &:= 2(\pi_{145} + \pi_{235}), \qquad & \widetilde{\rho}_2 &:= 2(\pi_{146} + \pi_{236}).
\end{aligned} \tag{2.9}$$

Recall from table 1 that before the orientifold projection there are in addition 5 Kähler moduli from the untwisted sector.

Next, we need to take a closer look at the moduli coming from the twisted sectors. The Θ^1- and the Θ^3-twisted sectors feature 16 $\mathbb{Z}_4$ fixed points, giving rise to 16 additional Kähler moduli. The Θ^2 action leaves the third torus invariant, but acts nontrivially on the first two. Of the sixteen $\mathbb{Z}_2$ fixed points there are four that are also fixed points under the $\mathbb{Z}_4$-action. To each of the sixteen fixed points we associate an exceptional 2-cycle $e_{\alpha\beta}$, $\alpha, \beta \in \{1,2,3,4\}$, where $\alpha = 1,4$ denote the $\mathbb{Z}_4$-invariant fixed points and $\alpha = 2,3$ denote the $\mathbb{Z}_2$-invariant fixed points that get mapped into each other under Θ (cf. figure 2). These give a total of 10 Kähler moduli. Certain linear combinations of these 2-cycles may be combined with the fundamental 1-cycles $\pi_{5,6}$ on the third torus to yield exceptional 3-cycles of topology $S^2 \times S^1$. Demanding invariance of the exceptional 3-cycles under the action of Θ and Θ^3, which is given by [8]

$$\Theta(e_{\alpha\beta} \otimes \pi_{5,6}) = \Theta^3(e_{\alpha\beta} \otimes \pi_{5,6}) = -e_{\eta(\alpha)\eta(\beta)} \otimes \pi_{5,6}, \tag{2.10}$$

with

$$\eta(1) = 1,\ \eta(4) = 4,\ \eta(2) = 3,\ \eta(3) = 2, \tag{2.11}$$

[8]The two $\mathbb{Z}_2$ fixed points are interchanged under Θ and Θ^3, while the $\mathbb{Z}_4$ fixed points are invariant (cf. figure 2). The minus sign in (2.10) stems from the reflection of the fundamental 1-cycle of the third torus.

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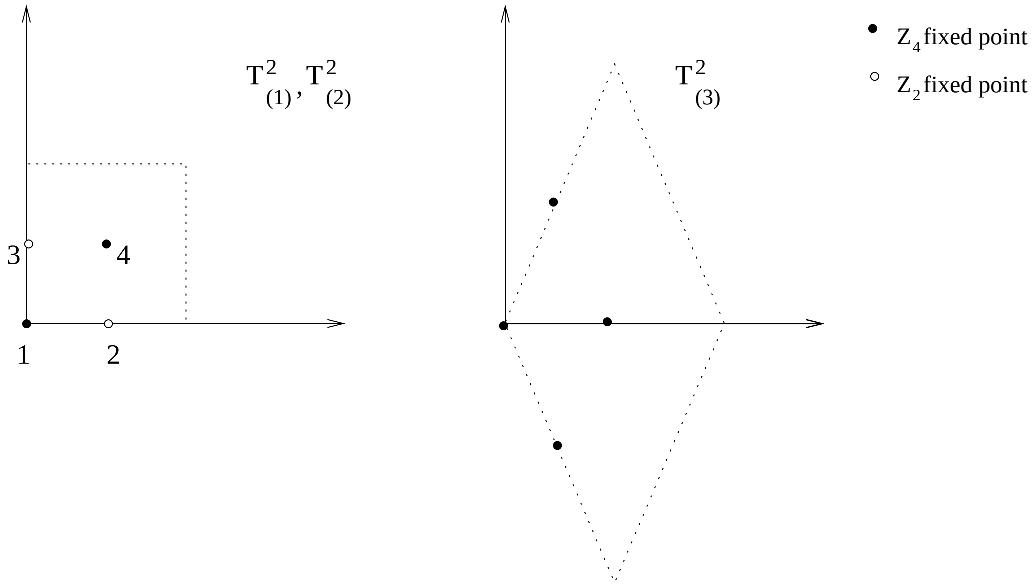

Figure 2: Fixed points of the first two tori and the third torus.

one finds precisely twelve invariant combinations,

$$\begin{aligned}
\epsilon_1 &:= (e_{12} - e_{13}) \otimes \pi_5, & \widetilde{\epsilon}_1 &:= (e_{12} - e_{13}) \otimes \pi_6, \\
\epsilon_2 &:= (e_{42} - e_{43}) \otimes \pi_5, & \widetilde{\epsilon}_2 &:= (e_{42} - e_{43}) \otimes \pi_6, \\
\epsilon_3 &:= (e_{21} - e_{31}) \otimes \pi_5, & \widetilde{\epsilon}_3 &:= (e_{21} - e_{31}) \otimes \pi_6, \\
\epsilon_4 &:= (e_{24} - e_{34}) \otimes \pi_5, & \widetilde{\epsilon}_4 &:= (e_{24} - e_{34}) \otimes \pi_6, \\
\epsilon_5 &:= (e_{22} - e_{33}) \otimes \pi_5, & \widetilde{\epsilon}_5 &:= (e_{22} - e_{33}) \otimes \pi_6, \\
\epsilon_6 &:= (e_{23} - e_{32}) \otimes \pi_5, & \widetilde{\epsilon}_6 &:= (e_{23} - e_{32}) \otimes \pi_6.
\end{aligned} \tag{2.12}$$

Kaluza-Klein reduction of type IIA theory. The low energy limit of type IIA superstring theory yields ten-dimensional type IIA supergravity. In order to cancel the $\hat{C}_{(7)}$-tadpole, it turns out to be convenient for our purposes to allow for a nonzero $\hat{F}_{(0)}$. This effectively leads to massive type IIA SUGRA with mass $m_0 = \hat{F}_{(0)}$. The corresponding action in the string frame is given by[9]

$$\begin{aligned}
S^{(10)}_{IIA,m_0} =& S_{\text{kin}} + S_{\text{CS}} + S_{O6} \\
=& \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-\hat{g}} \left(e^{-2\hat{\phi}} (\hat{R} + 4\partial_\mu \hat{\phi} \partial^\mu \hat{\phi} - \frac{1}{2} |\hat{H}_3^{\text{tot}}|^2) - (|\hat{F}_2|^2 + |\hat{F}_4|^2 + m_0^2) \right) \\
& - \frac{1}{2\kappa_{10}^2} \int \Big(\hat{B}_{(2)} \wedge d\hat{C}_{(3)} \wedge d\hat{C}_{(3)} + 2\hat{B}_{(2)} \wedge d\hat{C}_{(3)} \wedge \hat{F}^{\text{bg}}_{(4)} + \hat{C}_{(3)} \wedge \hat{H}^{\text{bg}}_{(3)} \wedge d\hat{C}_{(3)} \\
& \qquad - \frac{m_0}{3} \hat{B}_{(2)} \wedge \hat{B}_{(2)} \wedge \hat{B}_{(2)} \wedge d\hat{C}_{(3)} + \frac{m_0^2}{20} \hat{B}_{(2)} \wedge \hat{B}_{(2)} \wedge \hat{B}_{(2)} \wedge \hat{B}_{(2)} \wedge \hat{B}_{(2)} \Big)
\end{aligned} \tag{2.13}$$

[9] We use hats to indicate that a field is ten-dimensional, following the conventions of [32]. Note also that in our convention for the RR fields we have an additional factor of $\sqrt{2}$.

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$$+ 2\mu_6 \int_{O6} d^7\xi e^{-\hat{\phi}} \sqrt{-\hat{g}} \; - 2\sqrt{2}\mu_6 \int_{O6} \hat{C}_{(7)},$$

where $2\kappa_{10}^2 = (2\pi)^7\alpha'^4$, $\mu_6 = (2\pi)^{-6}\alpha'^{-7/2}$ and the field strengths are given by

$$\hat{H}^{\text{tot}}_{(3)} = d\hat{B}_{(2)} + \hat{H}^{\text{bg}}_{(3)}, \tag{2.14a}$$

$$\hat{F}_{(2)} = d\hat{C}_{(1)} + m_0\hat{B}_{(2)}, \tag{2.14b}$$

$$\hat{F}_{(4)} = d\hat{C}_{(3)} + \hat{F}^{\text{bg}}_{(4)} - \hat{C}_{(1)} \wedge \hat{H}^{\text{tot}}_{(3)} - \frac{m_0}{2}\hat{B}_{(2)} \wedge \hat{B}_{(2)}. \tag{2.14c}$$

In the framework of standard Kaluza-Klein reduction, we expand the ten-dimensional gauge potentials in terms of harmonic forms on the internal space $Y = T^6/\mathbb{Z}_4$, namely

$$\begin{aligned} &\hat{C}_{(1)} = A^0(x),\ \hat{B}_{(2)} = B_{(2)}(x) + b^A(x)\omega_A, \quad A = 1,\ldots,h^{(1,1)}, \\ &\hat{C}_{(3)} = C_{(3)}(x) + A^A(x)\wedge\omega_A + \xi^K(x)\alpha_K - \tilde{\xi}_K(x)\beta^K, \quad K = 0,\ldots,h^{(2,1)}. \end{aligned} \tag{2.15}$$

where $b^A, \xi^K, \tilde{\xi}_K$ are scalars in four dimensions, A^0, A^A are four-dimensional one-forms and $B_{(2)}$ and $C_{(3)}$ are four-dimensional two- and three-forms respectively. The harmonic $(1,1)$-forms ω_A form a basis of $H^{(1,1)}(Y)$ with dual $(2,2)$-forms $\tilde{\omega}_A$, which constitute a harmonic basis of $H^{(2,2)}(Y)$. Moreover, $(\alpha_K, \beta^L) \in H^{(3)}(Y)$ form a real, sympletic basis of harmonic 3-forms on Y with dimension $h^{(3)} = 2h^{(2,1)} + 2$. The intersection numbers are

$$\int_Y \alpha_K \wedge \beta^L = \delta_K^L, \quad \int_Y \omega_A \wedge \tilde{\omega}^B = \delta_A^B. \tag{2.16}$$

Details of the orientifold projection. After modding out by the orientifold projection $\mathcal{O}$, we will be left with an $\mathcal{N} = 1$ supergravity action. To determine the $\mathcal{O}$-invariant states, first recall that the ten-dimensional fields show the following behavior under $(-1)^{F_L}$ and Ω_p (for a review, cf. [34]),

$$(-1)^{F_L}: \ \text{odd}: \hat{C}_{(1)}, \hat{C}_{(3)}, \quad \text{even}: \hat{\phi}, \hat{g}, \hat{B}_{(2)}, \tag{2.17}$$

$$\Omega_p: \ \text{odd}: \hat{B}_{(2)}, \hat{C}_{(3)}, \quad \text{even}: \hat{\phi}, \hat{g}, \hat{C}_{(1)}. \tag{2.18}$$

Accordingly, states that are $\mathcal{O}$-invariant have to satisfy

$$\begin{aligned} &\sigma^*\hat{\phi} = +\hat{\phi},\ \sigma^*\hat{g} = +\hat{g},\ \sigma^*\hat{B}_{(2)} = -\hat{B}_{(2)}, \\ &\sigma^*\hat{C}_{(1)} = -\hat{C}_{(1)},\ \sigma^*\hat{C}_{(3)} = +\hat{C}_{(3)}. \end{aligned} \tag{2.19}$$

Therefore we want to investigate how the cohomology groups split into even and odd subspaces under the antiholomorphic involution σ,

$$H^p(Y) = H^p_+(Y) \oplus H^p_-(Y). \tag{2.20}$$

The relevant cohomology groups together with their basis elements are summarized in table 3.[10] Let us begin by studying the behavior of the $(1,1)$-forms in the untwisted sector.

[10] Note that the volume form on $T^6/\mathbb{Z}_4$ is odd under σ.

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cohomology group	$H_+^{(1,1)}$	$H_-^{(1,1)}$	$H_+^{(2,2)}$	$H_-^{(2,2)}$	$H_+^{(3)}$	$H_-^{(3)}$
dimension	$h_+^{(1,1)}$	$h_-^{(1,1)}$	$h_-^{(1,1)}$	$h_+^{(1,1)}$	$h^{(2,1)}+1$	$h^{(2,1)}+1$
basis	ω_α	ω_a	$\tilde{\omega}^a$	$\tilde{\omega}^\alpha$	a_K	b^K

Table 3: Cohomology groups and their basis elements.

We will discuss the twisted sector moduli in section 4. There are four σ-odd $\mathbb{Z}_4$-invariant (unnormalized) $(1,1)$-forms, namely

$$\sigma : (dz^i \wedge d\overline{z}^i) \mapsto -(dz^i \wedge d\overline{z}^i), \quad i = 1,2,3, \tag{2.21a}$$

$$\sigma : (dz^1 \wedge d\overline{z}^2 + e^{\mathrm{i}\pi/2} d\overline{z}^1 \wedge dz^2) \mapsto -(dz^1 \wedge d\overline{z}^2 + e^{\mathrm{i}\pi/2} d\overline{z}^1 \wedge dz^2) \tag{2.21b}$$

and one even $(1,1)$-form,

$$\sigma : (dz^1 \wedge d\overline{z}^2 - e^{\mathrm{i}\pi/2} d\overline{z}^1 \wedge dz^2) \mapsto +(dz^1 \wedge d\overline{z}^2 - e^{\mathrm{i}\pi/2} d\overline{z}^1 \wedge dz^2). \tag{2.22}$$

Consequently, $h^{(1,1)}_{+,untw.} = 1$ and $h^{(1,1)}_{-,untw.} = 4$. Moreover, we can combine the $\mathbb{Z}_4$-invariant $(2,1)$-form and the corresponding $(1,2)$-form into an even and an odd combination under σ,

$$\sigma : (dz^1 \wedge dz^2 \wedge d\overline{z}^3 \pm \mathrm{i} d\overline{z}^1 \wedge d\overline{z}^2 \wedge dz^3) \mapsto \pm(dz^1 \wedge dz^2 \wedge d\overline{z}^3 \pm \mathrm{i} d\overline{z}^1 \wedge d\overline{z}^2 \wedge dz^3). \tag{2.23}$$

Fluxes. The following background fluxes of the NS-NS and R-R field strengths are consistent with the orientifold projection and may thus be turned on:

$$\hat{F}_0^{\mathrm{bg}} = m_0,\ \hat{F}_2^{\mathrm{bg}} = -m_a\omega_a,\ \hat{F}_4^{\mathrm{bg}} = e_a\tilde{\omega}^a,\ \hat{H}_3^{\mathrm{bg}} = -p_K b^K, \tag{2.24}$$

where we have taken into account the appropriate behavior of the fluxes under σ. The indices $a = 1, \ldots, h^{(1,1)}_{-,untw.} = 4$ and $K = 0, \ldots, h^{(2,1)}_{untw.} = 1$ label the basis elements of the cohomology groups, as given in table 3, but are restricted to the untwisted sector. More explicitly, we have

$$\omega_1 = \left(\frac{\kappa}{2}\right)^{1/3} \mathrm{i} dz^1 \wedge d\overline{z}^1, \tag{2.25a}$$

$$\omega_2 = \left(\frac{\kappa}{2}\right)^{1/3} \mathrm{i} dz^2 \wedge d\overline{z}^2, \tag{2.25b}$$

$$\omega_3 = \left(\frac{\kappa}{2}\right)^{1/3} \frac{1}{U_2} \mathrm{i} dz^3 \wedge d\overline{z}^3, \tag{2.25c}$$

$$\omega_4 = \left(\frac{\kappa}{2}\right)^{1/3} \frac{(1-\mathrm{i})}{2} (dz^1 \wedge d\overline{z}^2 - \mathrm{i} dz^2 \wedge d\overline{z}^1), \tag{2.25d}$$

and in addition,

$$\tilde{\omega}^1 = \left(\frac{1}{(4\kappa)^{1/3} U_2}\right) (\mathrm{i} dz^2 \wedge d\overline{z}^2) \wedge (\mathrm{i} dz^3 \wedge d\overline{z}^3), \tag{2.26a}$$

$$\tilde{\omega}^2 = \left(\frac{1}{(4\kappa)^{1/3} U_2}\right) (\mathrm{i} dz^3 \wedge d\overline{z}^3) \wedge (\mathrm{i} dz^1 \wedge d\overline{z}^1), \tag{2.26b}$$

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$$\tilde{\omega}^3 = \left(\frac{1}{(4\kappa)^{1/3}}\right)(\mathrm{i}dz^1 \wedge d\overline{z}^1) \wedge (\mathrm{i}dz^2 \wedge d\overline{z}^2), \tag{2.26c}$$

$$\tilde{\omega}^4 = -\left(\frac{1}{(4\kappa)^{1/3}U_2}\right)\frac{(1-\mathrm{i})}{2}(dz^1 \wedge d\overline{z}^2 - \mathrm{i}dz^2 \wedge d\overline{z}^1) \wedge (\mathrm{i}dz^3 \wedge d\overline{z}^3), \tag{2.26d}$$

such that

$$\int_Y \omega_1 \wedge \omega_2 \wedge \omega_3 = -\int_Y \omega_3 \wedge \omega_4 \wedge \omega_4 = \kappa \tag{2.27}$$

and

$$\int_Y \omega_a \wedge \tilde{\omega}^b = \delta_a^b. \tag{2.28}$$

We normalize the volume form such that

$$\mathrm{i}\int_Y \Omega \wedge \overline{\Omega} = 1 \implies \Omega = \frac{(1-\mathrm{i})}{2\sqrt{U_2}}dz^1 \wedge dz^2 \wedge dz^3, \tag{2.29}$$

and choose our three forms to be

$$a_0 = \frac{1}{2}(dx^1 \wedge dx^2 - dy^1 \wedge dy^2 + dx^1 \wedge dy^2 + dy^1 \wedge dx^2) \wedge dx^3, \tag{2.30a}$$

$$a_1 = \frac{1}{4}(dx^1 \wedge dx^2 - dy^1 \wedge dy^2 - dx^1 \wedge dy^2 - dy^1 \wedge dx^2) \wedge dy^3, \tag{2.30b}$$

$$b_0 = 2(dx^1 \wedge dx^2 - dy^1 \wedge dy^2 + dx^1 \wedge dy^2 + dy^1 \wedge dx^2) \wedge dy^3, \tag{2.30c}$$

$$b_1 = -4(dx^1 \wedge dx^2 - dy^1 \wedge dy^2 - dx^1 \wedge dy^2 - dy^1 \wedge dx^2) \wedge dx^3. \tag{2.30d}$$

Ω is given in this basis by

$$\Omega = \frac{1}{\sqrt{U_2}}\,a_0 + 2\sqrt{U_2}\,a_1 + \mathrm{i}\frac{\sqrt{U_2}}{4}\,b_0 + \mathrm{i}\frac{1}{8\sqrt{U_2}}\,b_1. \tag{2.31}$$

The mixed-index part of the metric will be parameterized in the following way,

$$g_{i\bar{j}} = \begin{pmatrix} \gamma_1 & \gamma_4 + \mathrm{i}\gamma_5 & 0 \\ \gamma_4 - \mathrm{i}\gamma_5 & \gamma_2 & 0 \\ 0 & 0 & \gamma_3 \end{pmatrix}. \tag{2.32}$$

Taking into account the action of σ on g, one finds that $g_{1\bar{2}} = \mathrm{i}g_{2\bar{1}}$, so that $\gamma_4 = \gamma_5$. Therefore, one Kähler modulus of the untwisted sector gets projected by the orientifold.[11]

3. Moduli stabilization

We are now ready to calculate the potential for the various moduli fields discussed above. In the next subsection, we will directly calculate the potential from the (massive) IIA supergravity action compactified on the orientifold in the presence of fluxes. Moreover, we will derive several conditions, such as a tadpole cancelation condition and another condition on the 3-form axions ξ^0 and ξ^1 which are related to the complex structure.

[11] Note that there is a non-vanishing metric component of pure type, namely

$$\delta g_{\bar{3}\bar{3}} = -\frac{1}{||\Omega||^2}\overline{\Omega}_{\bar{3}}{}^{kl}(\chi_K)_{kl\bar{3}}(\tilde{z}^K), \tag{2.33}$$

corresponding to the deformations of the complex structure. In our conventions, the untwisted complex structure modulus U_2 also shows up in the effective potential for the untwisted Kähler moduli below.

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3.1 Dimensional (Kaluza-Klein) reduction from 10 to 4 dimensions

Again, we shall first restrict ourselves to the untwisted sector of the orientifold model.

Quantization of fluxes. We impose the usual cohomological quantization condition for a canonically normalized field strength,

$$\int \hat{F}_p = 2\kappa_{10}^2 \mu_{8-p} f_p = (2\pi)^{p-1} \alpha'^{(p-1)/2} f_p. \tag{3.1}$$

Accordingly, we have[12]

$$m_0 = \frac{f_0}{2\sqrt{2}\pi\sqrt{\alpha'}},\ m_a = \frac{2\pi\sqrt{\alpha'} f_2^{(a)}}{\sqrt{2}},\ p_K = (2\pi)^2 \alpha' h_3^{(K)},\ e_a = \frac{\kappa^{1/3}}{\sqrt{2}} (2\pi\sqrt{\alpha'})^3 f_4^{(a)}, \tag{3.2}$$

where $f_0, f_2^{(a)}, h_3^{(K)}, f_4^{(a)} \in \mathbb{Z}$.

Tadpole cancelation conditions. The $O6$-plane will generate a tadpole for the $\hat{C}_7$-potential, which we want to cancel solely by background fluxes without adding $D6$-branes. Noting that $*\hat{F}_{(2)} = d\hat{C}_7 - \hat{C}_5 \wedge \hat{H}_3 - \frac{m_0}{24} \hat{B}_2 \wedge \hat{B}_2 \wedge \hat{B}_2 \wedge \hat{B}_2$ contains $\hat{C}_7$, the integrated equations of motion for the $\hat{C}_7$-potential yield

$$\int d\hat{F}_{(2)} = \int m_0 \hat{H}_3^{\text{bg}} \stackrel{!}{=} 2\sqrt{2}\kappa_{10}^2 \mu_6 = 2(\sqrt{2}\pi\sqrt{\alpha'}). \tag{3.3}$$

The $O6$-plane wraps each of the cycles $[a_0] = (\rho_1 + \rho_2)$ and $[a_1] = (2(\tilde{\rho}_1 - \tilde{\rho}_2) + \rho_2 - \rho_1)$ once. Thus we have to integrate (3.3) over $[b_K], K = 0, 1$ leading to

$$m_0 p_K = -2(\sqrt{2}\pi\sqrt{\alpha'}), \quad K = 0, 1. \tag{3.4}$$

Taking into account the quantization condition (3.1), we arrive at the tadpole cancelation conditions

$$\begin{aligned} m_0 p_0 = m_0 p_1 = (\sqrt{2}\pi\sqrt{\alpha'}) f_0 h_3^{(K)} &= -2(\sqrt{2}\pi\sqrt{\alpha'}) \\ \Rightarrow (f_0, h_3^{(K)}) &= \pm(2, -1) \text{ or } \pm(1, -2). \end{aligned} \tag{3.5}$$

For later convenience we define $p \equiv p_0 = p_1$.

Potential for the untwisted complex structure axion. We will begin our discussion of the complex structure moduli by considering the associated axions first. A more detailed examination of the complex structure deformations will be carried out in the next subsection. It actually turns out that the contribution to the superpotential coming from $\hat{H}_{(3)}^{\text{bg}}$ fixes the real part of the complex structure hypermultiplet (namely the geometric complex structure moduli), while it leaves the imaginary part (the axions) unfixed. After the orientifold projection, the remaining axionic modes are[13]

$$\hat{C}_{(3)} = \xi^0 a_0 + \xi^1 a_1, \tag{3.6}$$

[12] Note the additional factor of $\sqrt{2}$ for the RR fields in our conventions.

[13] We have chosen a symplectic basis for $H^{(3)}(Y)$ such that all the a_K are σ-even and all the b^K are σ-odd.

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noting that $\hat{C}_{(3)}$ has to be even under the involution σ in our construction. The discussion here mostly parallels [22]. The RR field $\hat{C}_{(3)}$ only appears in the Chern-Simons piece of the massive IIA SUGRA action (2.13). It is important to notice that $\hat{C}_{(3)} \wedge \hat{H}^{\rm bg}_{(3)} \wedge d\hat{C}_{(3)}$ is nonvanishing only if $d\hat{C}_{(3)}$ is polarized in the noncompact directions. Since it does not contain physical degrees of freedom, we will treat it as a Lagrange multiplier $\mathcal{F}_0 := dC_{(3)}$. Plugging its equation of motion back into the action yields

$$S_{\mathcal{F}_0} = -\frac{1}{2\kappa_{10}^2} \int \mathcal{F}_0 \wedge *\mathcal{F}_0. \tag{3.7}$$

Minimizing this contribution to the potential is tantamount to setting $\mathcal{F}_0 = 0$. Doing this and integrating over Y results in an equation involving the 3-form axions, namely

$$p_0 \xi^0 + p_1 \xi^1 = e_0 + e_a b_a - \kappa m_0 b_3 (b_1 b_2 - \frac{b_4^2}{2}), \tag{3.8}$$

with the definition $e_0 := \int \hat{F}^{\rm bg}_{(6)}$. This means that only one linear combination of the axions is fixed while there is another (independent) one that remains unfixed. This is consistent with the results obtained below from analyzing the superpotential. One could either try to stabilize the remaining axion by introducing nonperturbative effects such as Euclidean $D2$-instantons or by using the unfixed axion(s) to give mass to (potentially anomalous) U(1) brane fields via the Stückelberg mechanism [24].

Equations of motions for the b_a. For simplicity we will set[14] $\hat{F}_2^{bg} = 0$. Since $\hat{C}_1$ has no zero modes, the contributions from the $|\hat{F}_2|^2$ and $|\hat{F}_4|^2$ terms in the action are at least quadratic in the b_a. Since the Chern-Simons term linear in $\hat{B}_2$ has been taken into account above, we find that the action contains no terms linear in b_a. Therefore there is a solution with $b_a = 0, \forall a$. Since we will find supersymmetric and non-supersymmetric vacua some of these solutions might have instabilities. We will further investigate this at the end of this section.

Flux generated potential for the untwisted Kähler and complex structure moduli. In this section we will stabilize the remaining untwisted moduli. We will work in the four dimensional Einstein frame, so we define $g_{(4)\mu\nu} = \frac{e^{\hat{\phi}}}{\sqrt{vol_{(6)}}} g^E_{(4)\mu\nu}$. The effective potential is defined as

$$S = \frac{1}{\kappa_{10}^2} \int d^4x \sqrt{-g^E_{(4)}} (-V_{\rm eff}). \tag{3.9}$$

For $b_a = 0$ and the ξ^K satisfying their equation of motion we only get contributions from the terms $|\hat{H}_3^{\rm tot}|^2, |\hat{F}_4|^2, m_0^2$ and the $O6$ Born-Infeld piece. They are

$$\begin{aligned} V_{\rm eff} =& \frac{e^{2\hat{\phi}}}{vol_{(6)}^2} p^2 (\frac{1}{U_2} + 4U_2) + \frac{e^{4\hat{\phi}}}{2vol_{(6)}^3} \\ & \times \left[\sum_{i=1}^3 e_i^2 v_i^2 + e_4^2 (v_1 v_2 + \frac{v_4^2}{2}) + e_1 e_2 v_4^2 + 2 e_4 v_4 (e_1 v_1 + e_2 v_2) \right] \end{aligned} \tag{3.10}$$

[14]Solutions with $\hat{F}_2^{bg} \neq 0$ have qualitatively the same behavior as the $\hat{F}_2^{bg} = 0$ solution as will be shown later.

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$$+\frac{m_0^2}{2}\frac{e^{4\hat{\phi}}}{vol_{(6)}}-2|m_0p|\frac{e^{3\hat{\phi}}}{vol_{(6)}^{3/2}}\left(\frac{1}{\sqrt{U_2}}+2\sqrt{U_2}\right), \tag{3.11}$$

where

$$\begin{aligned}
v_1 &= \frac{1}{2}\left(\frac{2}{\kappa}\right)^{1/3}\gamma_1, \qquad & v_2 &= \frac{1}{2}\left(\frac{2}{\kappa}\right)^{1/3}\gamma_2,\\
v_3 &= \frac{1}{2}\left(\frac{2}{\kappa}\right)^{1/3}U_2\,\gamma_3, \qquad & v_4 &= -\left(\frac{2}{\kappa}\right)^{1/3}\gamma_4,
\end{aligned}$$

$$\begin{aligned}
vol_{(6)} &= \int_Y dx^1\wedge dy^1\wedge dx^2\wedge dy^2\wedge dx^3\wedge dy^3\sqrt{g_{(6)}} = U_2\frac{\gamma_3\left(\gamma_1\gamma_2-2\gamma_4^2\right)}{4}\\
&= \kappa v_3\left(v_1v_2-\frac{v_4^2}{2}\right).
\end{aligned}$$

Extremizing the potential with respect to the complex structure U_2 fixes it at

$$U_2=\frac{1}{2}. \tag{3.12}$$

Now we solve

$$v_a\frac{\partial V}{\partial v_a}+\frac{7}{4}\frac{\partial V}{\partial\hat{\phi}}=0, \tag{3.13}$$

and find

$$e^{\hat{\phi}}\sqrt{vol_{(6)}}=\frac{5}{\sqrt{2}}\left|\frac{p}{m_0}\right|. \tag{3.14}$$

This is (almost) fixed by the tadpole cancelation conditions, cf. equation (3.5) above. This condition ensures that, for minima of the potential, the string coupling automatically becomes small if we tune the fluxes such that the internal volume becomes large enough to trust the supergravity approximation we are using. Relation (3.14) can be used to eliminate the dilaton dependence of the potential. Once the minima have been found, said relation fixes the dilaton w.r.t. a specific set of fluxes. The potential simplifies to

$$\begin{aligned}
V_{\text{eff}} = \frac{25}{8}\frac{p^4}{m_0^2}\Bigg(\frac{-39}{vol_{(6)}^3}+\frac{25}{m_0^2\,vol_{(6)}^5}\Bigg[&\sum_{i=1}^{3}e_i^2v_i^2+e_4^2(v_1v_2+\frac{v_4^2}{2})+e_1e_2v_4^2\\
&+2e_4v_4(e_1v_1+e_2v_2)\Bigg]\Bigg).
\end{aligned}$$

It now only depends on the Kähler moduli v_1, v_2, v_3, v_4. Extremizing with respect to all of the Kähler moduli leads to five sets of solutions. The first is

$$\begin{aligned}
v_1 &= \pm e_2\sqrt{\frac{10}{3}}\sqrt{\left|\frac{e_3}{\kappa m_0(2e_1e_2-e_4^2)}\right|},\\
v_2 &= \pm e_1\sqrt{\frac{10}{3}}\sqrt{\left|\frac{e_3}{\kappa m_0(2e_1e_2-e_4^2)}\right|},\\
v_3 &= \sqrt{\frac{5}{6}}\sqrt{\left|\frac{2e_1e_2-e_4^2}{\kappa m_0e_3}\right|},
\end{aligned} \tag{3.15}$$

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$$v_4 = \mp e_4\sqrt{\frac{10}{3}}\sqrt{\left|\frac{e_3}{\kappa m_0(2e_1e_2 - e_4^2)}\right|}.$$

As we will see below this solution encompasses the supersymmetric solution obtained from minimizing the potential of the 4-d SUGRA action. To allow for a geometrical interpretation of the solution we have to demand that the volume $vol_{(6)}$ and v_3 the area of the third torus are bigger than zero. This implies that $(2e_1e_2 - e_4^2) > 0$ which requires $\mathrm{sign}[e_1e_2] > 0$. The volume is

$$vol_{(6)} = \frac{5}{3}\sqrt{\frac{5}{6}}\sqrt{\left|\frac{e_3(2e_1e_2 - e_4^2)}{\kappa m_0^3}\right|}. \tag{3.16}$$

It can be made parametrically large by tuning the fluxes to large values. The string coupling is determined to be

$$g_s = e^{\hat{\phi}} = |p|\left(\frac{135}{2}\left|\frac{\kappa}{m_0e_3(2e_1e_2 - e_4^2)}\right|\right)^{1/4}. \tag{3.17}$$

Thus, there is a (countably) infinite number of vacua with small string coupling and large volume.[15]

The value of the potential at the minimum is

$$V_{\min} = -\frac{243}{25}\sqrt{\frac{6}{5}}\sqrt{\left|\frac{\kappa^3 m_0^5}{(e_3(2e_1e_2 - e_4^2))^3}\right|}\, p^4, \tag{3.18}$$

which is always negative so that the vacua are anti-de-Sitter.

The second set of solutions is

$$\begin{aligned} v_1 &= \pm e_4\sqrt{\frac{5}{3}}\sqrt{\left|\frac{e_2e_3}{\kappa m_0 e_1(2e_1e_2 - e_4^2)}\right|}, \\ v_2 &= \pm e_4\sqrt{\frac{5}{3}}\sqrt{\left|\frac{e_1e_3}{\kappa m_0 e_2(2e_1e_2 - e_4^2)}\right|}, \\ v_3 &= \sqrt{\frac{5}{6}}\sqrt{\left|\frac{2e_1e_2 - e_4^2}{\kappa m_0 e_3}\right|}, \\ v_4 &= \mp 2\sqrt{\frac{5}{3}}\sqrt{\left|\frac{e_1e_2e_3}{\kappa m_0(2e_1e_2 - e_4^2)}\right|}. \end{aligned} \tag{3.19}$$

For this case we have to demand that $(2e_1e_2 - e_4^2) < 0$ and $\mathrm{sign}[e_1e_2] > 0$. The volume, the string coupling and the potential at the minimum are the same as above. This is also the case for all the other solutions.

The next set of solutions has v_4 fixed at zero

$$v_1 = \pm\sqrt{\frac{5}{3}}\sqrt{\left|\frac{e_2e_3}{\kappa m_0 e_1}\right|}, \tag{3.20}$$

[15] If we set for example $e_1 = e_2 = e_3 = e_4 \equiv e \to \infty$, we have $vol \sim e^{3/2}$, $\quad e^{\hat{\phi}} \sim e^{-3/4}$.

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$$
\begin{aligned}
v_2 &= \pm\sqrt{\frac{5}{3}}\sqrt{\left|\frac{e_1 e_3}{\kappa m_0 e_2}\right|},\\
v_3 &= \sqrt{\frac{5}{6}}\sqrt{\left|\frac{2e_1 e_2 - e_4^2}{\kappa m_0 e_3}\right|},\\
v_4 &= 0.
\end{aligned}
$$

It requires sign$[e_1 e_2] < 0$ which implies $(2e_1 e_2 - e_4^2) < 0$.

We furthermore find solutions in which one of the Kähler moduli is unstabilized

$$
\begin{aligned}
v_1 &= \frac{1}{e_1^2}\left((-e_1 e_2 + e_4^2)v_2 \pm e_4\sqrt{|2e_1 e_2 - e_4^2|v_2^2 - \left|\frac{10}{3}\frac{e_1^2 e_3}{\kappa m_0}\right|}\right), \qquad (3.21)\\
v_2 &= \textbf{unfixed},\\
v_3 &= \sqrt{\frac{5}{6}}\sqrt{\left|\frac{2e_1 e_2 - e_4^2}{\kappa m_0 e_3}\right|},\\
v_4 &= \frac{1}{e_1}\left((-e_4 v_2 \mp \sqrt{|2e_1 e_2 - e_4^2|v_2^2 - \left|\frac{10}{3}\frac{e_1^2 e_3}{\kappa m_0}\right|}\right).
\end{aligned}
$$

These solutions require $(2e_1 e_2 - e_4^2) < 0$ and $v_2^2 > \left|\frac{10}{3}\frac{e_1^2 e_3}{\kappa m_0(2e_1 e_2 - e_4^2)}\right|$. Since the action is invariant under the simultaneous exchange of $e_1 \leftrightarrow e_2$ and $v_1 \leftrightarrow v_2$, we have corresponding solutions in which v_1 is unfixed.

Although we have turned on the most generic fluxes compatible with the orbifold and orientifold projection, we found solutions that have one unstabilized geometric modulus. As we will see below these solutions are not supersymmetric.

Stability analysis for the b_a. Since we have found vacua that are non-supersymmetric, we have to check that our $b_a = 0$ solution is in fact stable. To do this we consider the terms quadratic in b_a and ξ^K [16]. We find

$$
\begin{aligned}
S_{axion} = \frac{1}{2\kappa_{10}^2}\int d^4x\sqrt{-g_4^E} \qquad (3.22)\\
\times\Bigg[-\frac{1}{2vol_{(6)}}\partial_\mu b^a\partial^\mu b^b\int_Y(\omega_a\wedge *_6\,\omega_b) - e^{2D}\partial_\mu\xi^K\partial^\mu\xi^L\int_Y(a_K\wedge *_6\,a_L)\\
- e^{4D}\Bigg(m_0^2 b^a b^b\int_Y(\omega_a\wedge *_6\,\omega_b) - m_0 b^a b^b e_c\int_Y(\omega_a\wedge\omega_b\wedge *_6\,\tilde{\omega}^c)\\
+\frac{(-p_K\xi^K + e_a b^a)^2}{vol_{(6)}}\Bigg)\Bigg],
\end{aligned}
$$

where we defined the four dimensional dilation as $e^D = \frac{e^{\hat{\phi}}}{\sqrt{vol_{(6)}}}$. Now one has to diagonalize the kinetic energy terms and calculate the mass-squared matrix (Hessian) for each of the solutions described above. To carry out the calculations in full generality is rather tedious.

[16]Remember that (3.8) implies that there is a mixing between the b_a and ξ^K.

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From the action we see that the result will depend on the explicit choices for the fluxes m_0 and e_a. We have calculated the mass-squared matrix for simple sets of fluxes for all of our vacua. In each case, we obtain positive mass eigenvalues with the exception of one zero eigenvalue corresponding to the unstabilized axion $\xi_0 - \xi_1$ (cf. (3.8)). Thus, there exists a stable solution for all vacua (with large fluxes). In conclusion, we see that the solution corresponding to $b_a = 0$, $\forall a$, is a stable minimum of the effective four-dimensional potential, at least for simple choices of the fluxes.

3.2 Effective $\mathcal{N} = 1$ SUGRA in $D = 4$

In this subsection we will analyze the problem from the point of view of the effective $\mathcal{N} = 1$ SUGRA theory in four dimensions. One of the virtues of working in this framework is that the untwisted and the twisted moduli can be treated on equal footing. As pointed out in [22], another advantage lies in the fact that this type of analysis can be used for general backgrounds since e.g., backreaction and worldsheet instanton corrections are naturally described in terms of the four-dimensional effective theory, whereas they cannot be described in terms of ten-dimensional supergravity. Based on the flux-generated superpotential, as worked out by Grimm and Louis [32] (see also [34]), we will analyze the F-flatness conditions $D_I W = 0$, where I runs over all moduli fields and $D_I = \partial_I + (\partial_I K)$ is the Kähler covariant derivative. Solutions to these equations correspond to supersymmetric minima of the scalar potential,

$$V = e^K \left(\sum_{I\bar{J}} G^{I\bar{J}} D_I W \overline{D_J W} - 3|W|^2 \right) + m_0 e^{K^Q} \mathrm{Im} W^Q, \tag{3.23}$$

namely

$$D_I W = 0 \Rightarrow dV = 0. \tag{3.24}$$

The opposite direction is not true. The structure of the Kähler potential $K = K^K + K^Q$ and the superpotential $W = W^K + W^Q$ will be discussed below.

$\mathcal{N} = 2$ SUGRA in $D = 4$. The dimensional reduction of (massive) type IIA supergravity from $D = 10$ to $D = 4$ on a Calabi-Yau manifold gives rise to $\mathcal{N} = 2$ supergravity in $D = 4$. The existence of one covariantly constant spinor on the internal CY (with SU(3) holonomy) ensures that there are two four-dimensional SUSY parameters; the compactification therefore preserves eight supercharges, hence $\mathcal{N} = 2$ in $D = 4$. In the presence of fluxes, the resulting effective theory in four dimensions is gauged, i.e., the hypermultiplets are charged under some of the vectormultiplets. For this to be consistent, the metric on the scalar manifold coordinatized by the hypermultiplets, which is in fact a quaternionic manifold, must possess isometries that in turn can be gauged. Table 4 lists the bosonic components of all $\mathcal{N} = 2$ multiplets. There are massless modes coming from deformations of the metric g of the CY manifold that respect the Ricci flatness condition $R_{mn} = 0$. This forces δg to satisfy the Lichnerowicz equation, whose solutions in our case can be identified with harmonic (1,1)- and (2,1)-forms on Y, corresponding to Kähler structure and complex structure deformations, respectively.

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gravity multiplet	1	$(g_{\mu\nu}, A^0)$
vectormultiplets	$h^{(1,1)}$	(A^A, v^A, b^A)
hypermultiplets	$h^{(2,1)}$	$(z^K, \xi^K, \tilde{\xi}_K)$
tensor multiplet	1	$(B_{(2)}, \hat{\phi}, \xi^0, \tilde{\xi}_0)$

Table 4: Bosonic part of the $\mathcal{N} = 2$ multiplets for Type IIA SUGRA on a CY3.

Kähler moduli space. Deformations of the Kähler form can be expanded in a basis of harmonic (1,1)-forms,

$$g_{i\bar{j}} + \delta g_{i\bar{j}} = -\mathrm{i}J_{i\bar{j}} = -\mathrm{i}v^A(\omega_A)_{i\bar{j}}, \quad A = 1, \dots, h^{(1,1)}. \tag{3.25}$$

These deformations can be supplemented by the $h^{(1,1)}$ real scalar fields $b^A(x)$ from the expansion of the B-field, yielding complex fields

$$t^A = b^A + \mathrm{i}v^A, \tag{3.26}$$

that parametrize the complexified Kähler cone. The moduli space of the complexified Kähler structure deformations $\mathcal{M}^{\mathrm{ks}}$ is a special Kähler manifold which can be seen by noting that the metric is given by

$$G_{\mathrm{AB}} = \frac{3}{2\kappa}\int_Y \omega_A \wedge *\omega_B = -\frac{3}{2}\left(\frac{\kappa_{\mathrm{AB}}}{\kappa} - \frac{3}{2}\frac{\kappa_A\kappa_B}{\kappa^2}\right) = \partial_{t^A}\partial_{t^B}K^{\mathrm{ks}}, \tag{3.27}$$

where the intersection numbers are defined as follows

$$\kappa = \int_Y J \wedge J \wedge J = \kappa_{\mathrm{ABC}}v^Av^Bv^C, \quad \kappa_A = \int_Y \omega_A \wedge J \wedge J = \kappa_{\mathrm{ABC}}v^Bv^C,$$
$$\kappa_{\mathrm{AB}} = \int_Y \omega_A \wedge \omega_B \wedge J = \kappa_{\mathrm{ABC}}v^C, \quad \kappa_{\mathrm{ABC}} = \int_Y \omega_A \wedge \omega_B \wedge \omega_C.$$

The Kähler potential for the Kähler structure deformations,

$$K^{\mathrm{ks}} = -\ln\left(\frac{\mathrm{i}}{6}\kappa_{\mathrm{ABC}}(t-\bar{t})^A(t-\bar{t})^B(t-\bar{t})^C\right) = -\ln\frac{4}{3}\kappa, \tag{3.28}$$

can be derived from a single holomorphic prepotential $\mathcal{G}(t) = -\frac{1}{6}\kappa_{\mathrm{ABC}}t^At^Bt^C$.

Complex structure moduli space. Complex structure deformations are associated with harmonic (1,2)-forms and are parametrized by complex fields $\tilde{z}^K$, $K = 1, \dots, h^{(2,1)}$, in the following way,

$$\delta g_{ij} = \frac{\mathrm{i}}{||\Omega||^2}\bar{\tilde{z}}^K(\overline{\chi}_K)_{i\bar{i}\bar{j}}\Omega^{\bar{i}\bar{j}}{}_j, \tag{3.29}$$

where the χ_K form a harmonic basis of $H^{(2,1)}(Y)$ and $||\Omega||^2 = \frac{1}{3!}\Omega_{ijk}\Omega^{ijk}$. The metric on the complex structure moduli space $\mathcal{M}^{\mathrm{cs}}$ is given by

$$G_{K\bar{L}} = -\frac{\int_Y \chi_K \wedge \overline{\chi}_L}{\int_Y \Omega \wedge \overline{\Omega}}. \tag{3.30}$$

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Kodaira's formula connects the χ_K to the variation of the harmonic (3,0)-form via

$$\chi_K(\tilde{z}, \overline{\tilde{z}}) = \partial_{\tilde{z}^K}\Omega(\tilde{z}) + \Omega(\tilde{z})\partial_{\tilde{z}^K}K^{\rm cs}, \tag{3.31}$$

where

$$K^{\rm cs}(\tilde{z}, \overline{\tilde{z}}) = -\ln\left[\mathrm{i}\int_Y \Omega\wedge\overline{\Omega}\right] = -\ln \mathrm{i}\left[\overline{Z}^K\mathcal{F}_K - Z^K\overline{\mathcal{F}}_K\right]. \tag{3.32}$$

Note that $G_{K\bar{L}} = \partial_{\tilde{z}^K}\partial_{\bar{\tilde{z}}^L}K^{\rm cs}$, thus proving that that $\mathcal{M}^{\rm cs}$ is a Kähler manifold. The holomorphic periods $Z^K, \mathcal{F}_K$ are the expansion coefficients of

$$\Omega = Z^K\alpha_K - \mathcal{F}_K\beta^K, \tag{3.33}$$

so that we have

$$Z^K = \int_Y \Omega\wedge\beta^K, \quad \mathcal{F}_K = \int_Y \Omega\wedge\alpha_K. \tag{3.34}$$

In fact, Ω is only defined up to a complex rescaling with a holomorphic function which changes the Kähler potential by a Kähler transformation. This symmetry can be used to fix a Kähler gauge, in which $Z^0 = 1$. The remaining periods can be identified with the $h^{(2,1)}$ complex structure deformations

$$\tilde{z}^K = \frac{Z^K}{Z^0}. \tag{3.35}$$

Moreover, we find that there exists a prepotential of which $\mathcal{F}_K$ is the first derivative, $\mathcal{F} = \frac{1}{2}Z^K\mathcal{F}_K$. This means that the metric $G_{K\bar{L}}$ is completely determined by $\mathcal{F}$. Therefore $\mathcal{M}^{\rm cs}$ is in fact a special Kähler manifold.

Supplementing the complex structure deformations $\tilde{z}^K$ with the corresponding axions ξ^K and $\tilde{\xi}_K$ from the RR 3-form $\hat{C}_3$ can be shown to result in a special quaternionic structure of the resulting moduli space. We will refer to this larger manifold, spanned by the scalars in the hypermultiplets, as $\mathcal{M}^Q$. In the next section we will use the fact that $\mathcal{M}^Q$ contains the special Kähler submanifold $\mathcal{M}^{\rm cs}$ spanned by the complex structure deformations.

Orientifold projection. As already mentioned above, the cohomology groups split into even and odd parts under the antiholomorphic involution σ (cf. (2.20)). The involution must act as [35]

$$\sigma^* J = -J, \quad \sigma^*\Omega = e^{2\mathrm{i}\theta}\overline{\Omega}. \tag{3.36}$$

The fixed loci of σ (which the $O6$-plane wraps) are special Lagrangian (sLag) 3-cycles Σ_n fulfilling

$$J\Big|_{\Sigma_n} = 0, \quad \mathrm{Im}(e^{-\mathrm{i}\theta}\Omega)\Big|_{\Sigma_n} = 0. \tag{3.37}$$

Together with the conditions (2.19) we are left with

$$J_c := B + \mathrm{i}J = \sum_{a=1}^{h_-^{(1,1)}} t^a\omega_a. \tag{3.38}$$

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multiplets	multiplicity	bosonic components
gravity multiplet	1	$g_{\mu\nu}$
vector multiplets	$h_+^{(1,1)}$	A^α
chiral multiplets	$h_-^{(1,1)}$	t^a
chiral multiplets	$h^{(2,1)}+1$	N^K

Table 5: $\mathcal{N}=1$ multiplets after orientifold projection.

Thus, the orientifold projection reduces the Kähler moduli space to a subspace without altering its complex structure and the Kähler potential is inherited directly from $\mathcal{N}=2$,

$$K^K(t^a) = -\log(\frac{4}{3}\kappa_{abc}v^a v^b v^c). \tag{3.39}$$

For the holomorphic (3,0)-form, we get

$$\Omega(\tilde{z}) = Z^K(\tilde{z})a_K - \mathcal{F}_K(\tilde{z})b^K, \tag{3.40}$$

where we have decomposed $H^{(3)}(Y) = H_+^{(3)}(Y) \oplus H_-^{(3)}(Y)$ as indicated in table 3. As remarked upon earlier, one can always perform a symplectic rotation on the resulting even and odd bases such that all a_K are even and all b^K are odd. Note that the $h_+^{(1,1)}$ vector multiplets do not contain any scalars and will therefore be disregarded. It is customary to package the remaining degrees of freedom in the following way,

$$\Omega_c = \hat{C}_{(3)} + 2\mathrm{iRe}(C\Omega), \tag{3.41}$$

where we have introduced the complex compensator $C = re^{-\mathrm{i}\theta}$, where $r = e^{-D+K^{\mathrm{cs}}/2}$. r transforms oppositely to the holomorphic 3-form under holomorphic transformations so as to render $C\Omega$ scale-invariant (the compensator replaces the irrelevant scale factor in favor of the physical dilaton field D; for more details see [32, 35]). The field $\hat{C}_{(3)} = \xi^K a_K$ comprises the surviving axionic modes. Finally, Ω_c can be expanded in a basis of $H_+^{(3)}(Y)$,

$$\Omega_c = 2N^K a_K, \tag{3.42}$$

where

$$N^K = \frac{1}{2}\int_Y \Omega_c \wedge b^K = \frac{1}{2}(\xi^K + 2\mathrm{iRe}(CZ^K)). \tag{3.43}$$

We have now reduced the number of moduli, while preserving the original $\mathcal{N}=2$ complex structure. Table 5 shows the surviving $\mathcal{N}=1$ spectrum. The $\mathcal{N}=1$ Kähler potential is given by

$$K^Q = -2\log\left(2\int \mathrm{Re}(C\Omega)\wedge *\mathrm{Re}(C\Omega)\right) = 4D, \tag{3.44}$$

where

$$\int \mathrm{Re}(C\Omega)\wedge *\mathrm{Re}(C\Omega) = -\mathrm{Re}(CZ^K)\mathrm{Im}(C\mathcal{F}_K) = \frac{e^{-2D}}{2}. \tag{3.45}$$

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For the four dimensional dilaton we have

$$e^D = \frac{e^{\hat{\phi}}}{\sqrt{vol}} = \sqrt{8} e^{\hat{\phi} + K^K/2}. \tag{3.46}$$

In conclusion, we have seen that from each quaternionic hypermultiplet only the real part of the complex structure modulus and one axion survives. The degrees of freedom in the universal hypermultiplet are also cut in half, namely the dilaton $\hat{\phi}$ and the axion ξ^0 survive.

3.3 Supersymmetric AdS vacua

It was demonstrated by Grimm and Louis [32] that dimensionally reducing massive type IIA supergravity from 10 to 4 dimensions, while neglecting the backreaction of the fluxes and other local sources on the geometry of the compactification manifold, leads to the following scalar potential,

$$V = e^{K^K + K^Q} \left(\sum_{I,J=t^a,N^K} G^{I\bar{J}} D_I W \overline{D_J W} - 3|W|^2 \right) + m_0 e^{K^Q} \mathrm{Im} W^Q. \tag{3.47}$$

The second term cancels with contributions from the $O6$-plane when the tadpole cancelation condition (3.3) is satisfied. The superpotential is given by

$$W(t^a, N^K) = W^Q(N^K) + W^K(t^a), \tag{3.48a}$$

$$W^Q(N^K) = \int_Y \Omega_c \wedge \hat{H}_{(3)} = -2 p_K N^K = -p_K \xi^K - 2\mathrm{i} p_K \mathrm{Re}(C Z^K), \tag{3.48b}$$

$$\begin{aligned} W^K(t^a) &= \int_Y e^{-J_c} \wedge \hat{F} = e_0 + \int_Y J_c \wedge \hat{F}_{(4)} - \frac{1}{2} \int_Y J_c \wedge J_c \wedge \hat{F}_{(2)} - \frac{m_0}{6} \int_Y J_c \wedge J_c \wedge J_c \\ &= e_0 + e_a t^a + \frac{1}{2} \kappa_{abc} t^a t^b m^c - \frac{m_0}{6} \kappa_{abc} t^a t^b t^c, \end{aligned} \tag{3.48c}$$

with the definition $\hat{F} = m_0 - \hat{F}^{\mathrm{bg}}_{(2)} - \hat{F}^{\mathrm{bg}}_{(4)} + \hat{F}^{\mathrm{bg}}_{(6)}$ (cf. (2.24)). In the following sections we will first analyze the equations for the moduli from the F-term conditions (3.24) in general and then specialize to the case at hand, namely the $T^6/\mathbb{Z}_4$ orientifold. It is important to note that these equations will be valid for all (untwisted and twisted) moduli. The discussion closely follows the one in [22].

Complex structure equations. Solving for $D_{N^K} W = 0$ yields

$$p_K + 2\mathrm{i} W \mathrm{Im}(C \mathcal{F}_K) e^{2D} = 0. \tag{3.49}$$

We shall study the real and imaginary parts of this equation separately. For the real part one gets

$$p_K - 2 e^{2D} \mathrm{Im}(W) \mathrm{Im}(C \mathcal{F}_K) = 0. \tag{3.50}$$

We immediately learn from this equation that $\mathrm{Im}(W) = 0$ is incompatible with non-vanishing $\hat{H}^{\mathrm{bg}}_{(3)}$-flux. Thus assuming $\mathrm{Im}(W) \neq 0$ we find that for each $p_{K_i} = 0$, we have $\mathrm{Im}(C\mathcal{F}_{K_i}) = 0$. For $p_{K_j} \neq 0$, one finds

$$e^{-K^{\mathrm{cs}}/2} \frac{p_{K_j}}{\mathrm{Im}(\mathcal{F}_{K_j})} = 2 e^D \mathrm{Im}(W) =: Q_0, \tag{3.51}$$

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thus fixing all geometric complex structure moduli (including the twisted ones, in our case $K = 0, \ldots, h^{(2,1)} = 7$). As noted above, these equations are invariant under rescalings of Ω and therefore do only depend on the $h^{(2,1)}$ inhomogeneous coordinates of $\mathcal{M}^{\rm cs}$, yielding $h^{(2,1)}$ equations for the $h^{(2,1)}$ moduli. The dilaton will be stabilized at

$$e^{-\hat{\phi}} = 4\sqrt{2}e^{K^K/2}\frac{\mathrm{Im}(W)}{Q_0}, \tag{3.52}$$

once complex structure and Kähler moduli are fixed.

Turning to the imaginary part of (3.49), we see that, due to the reality of the flux coefficients p_K, all K equations yield the same condition, namely (D and $C = r$ are real[17].)

$$2e^{2D}\mathrm{Re}(W)\mathrm{Im}(C\mathcal{F}_K) = 0 \Rightarrow \mathrm{Re}(W) = 0. \tag{3.53}$$

Comparing to the definition of W, this indeed gives the same condition on the axions as derived above (cf. (3.8)),

$$-p_K\xi^K + \mathrm{Re}(W^K) = 0, \tag{3.54}$$

where we have now correctly considered all the axions, including those from the twisted sectors. Another important observation can be made by multiplying (3.49) by $\mathrm{Re}(CZ_K)$ and summing over K. The resulting equation reads

$$-\mathrm{i}W = -p_K\mathrm{Re}(CZ^K) = \frac{1}{2}\mathrm{Im}(W^Q). \tag{3.55}$$

Now since $\mathrm{R}e(W) = 0$ (cf. (3.53)), we find

$$-\mathrm{i}W = \mathrm{Im}(W^K) + \mathrm{Im}(W^Q) = \frac{1}{2}\mathrm{Im}(W^Q) \Rightarrow \mathrm{Im}(W^Q) = -2\,\mathrm{Im}(W^K). \tag{3.56}$$

Therefore we can directly conclude that, provided the complex structure moduli are 'on-shell' (satisfy their equations of motion), the vacuum superpotential can be given solely in terms of the Kähler moduli, i.e.,

$$W(t^a, N^K) = -\mathrm{i}\,\mathrm{Im}(W^K(t^a)), \tag{3.57}$$

thus effectively decoupling the Kähler sector from the complex structure sector.

Kähler structure equations. Let us now consider the Kähler sector in more detail. The corresponding F-flatness conditions $D_{t^a}W = 0$ can be simplified making use of (3.57), yielding

$$\partial_{t^a}W^K - \mathrm{i}\partial_{t^a}K^K\mathrm{Im}(W^K) = 0. \tag{3.58}$$

The imaginary parts of these equations produce conditions on the B-field parameters b_a, due to the fact that K^K only depends on $v^a = \mathrm{Im}t^a$, ensuring the reality of the second term,

$$\mathrm{Im}\partial_{t^a}W^K = \kappa_{abc}v_b(m_c - m_0b_c) = 0. \tag{3.59}$$

[17]We absorb θ in the holomorphic 3-form so that it satisfies $\sigma^*\Omega = \overline{\Omega}$

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Therefore, b_c is stabilized at $b_c = \frac{m_c}{m_0}$ and vanishes when $\hat{F}^{\rm bg}_{(2)} = 0$, as claimed above. Of course, this assumes $m_0 \neq 0$ and also non-vanishing v_b and κ_{abc}. This leads us to the real part of equations (3.58). We will show that these yield $h_-^{(1,1)}$ equations to determine the $h_-^{(1,1)}$ moduli fields v^a or equivalently the γ^a used in the discussion earlier. They read

$$\mathrm{Re}(\partial_{t^a} W^K) + \mathrm{Im}(\partial_{t^a} K^K)\mathrm{Im}(W^K) = 0. \tag{3.60}$$

More explicitly, we have

$$(4e_a m_0 + 2\kappa_{apq} m^p m^q + 3m_0^2 \kappa_{apq} v^p v^q)\kappa_{def} v^d v^e v^f + (6m_0 e_d v^d + 3\kappa_{def} v^d m^e m^f)\kappa_{apq} v^p v^q = 0, \tag{3.61}$$

where we made frequent use of the equations for the b_a parameters (see above). Multiplying by v^a and summing over a leads to[18]

$$10 m_0 e_d v^d + 5\kappa_{def} v^d m^e m^f + 3m_0^2 \kappa_{def} v^d v^e v^f = 0. \tag{3.62}$$

This gives us one quadratic equation for every v_a, thus generically fixing all the Kähler structure moduli, namely

$$10 m_0 e_a + 5\kappa_{abc} m^b m^c + 3m_0^2 \kappa_{abc} v^b v^c = 0. \tag{3.63}$$

3.4 Application to the $T^6/\mathbb{Z}_4$ model

We start out by neglecting the twisted sector to show that we can reproduce the results found above. Then we discuss the details of the twisted sector and derive the results for all moduli.

Complex structure equations. Combining equations (3.51) and (2.31) we get[19]

$$-\frac{4p_0}{\sqrt{U_2}} = -8\sqrt{U_2}p_1 =: Q_0. \tag{3.64}$$

Assuming that we satisfy the tadpole cancelation conditions $p_0 = p_1 \equiv p$ implies that the complex structure is fixed at $U_2 = \frac{1}{2}$. Since $Q_0 = -4\sqrt{2}p$, the dilaton (cf. (3.52)) gets fixed at

$$e^{-\hat{\phi}} = -\frac{\sqrt{2}}{5}\frac{m_0}{p}\sqrt{vol_{(6)}}. \tag{3.65}$$

Note that this implies that $\mathrm{sign}[m_0 p] = -1$.
The axions as derived above in (3.54) satisfy

$$p_0\xi^0 + p_1\xi^1 = e_0 + e_a b_a + \frac{1}{2}\kappa_{abc} m_a(b_b b_c - v_b v_c) - \frac{m_0}{6}\kappa_{abc}(b_a b_b b_c - 3b_a v_b v_c), \tag{3.66}$$

which agrees with (3.8) for $b_a = \frac{m_a}{m_0}$.
Kähler structure equations. The equations (3.63) yield the following result for the untwisted Kähler moduli,

$$v_1 = \pm\sqrt{\frac{10}{3}}\frac{\hat{e}_2\sqrt{\hat{e}_3}}{\sqrt{\kappa m_0}\sqrt{-2\hat{e}_1\hat{e}_2 + \hat{e}_4^2}},$$

[18] Solving equation (3.61) directly gives no solution with $vol_{(6)} \neq 0$ and any of the $v_a = 0$.
[19] Recall that we have normalized Ω s.t. $\mathrm{i}\int \Omega \wedge \bar{\Omega} = 1$ so that $K^{\rm cs} = 0$.

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$$
\begin{aligned}
v_2 &= \pm\sqrt{\frac{10}{3}}\frac{\hat{e}_1\sqrt{\hat{e}_3}}{\sqrt{\kappa m_0}\sqrt{-2\hat{e}_1\hat{e}_2+\hat{e}_4^2}},\\
v_3 &= \mp\sqrt{\frac{5}{6}}\frac{\sqrt{-2\hat{e}_1\hat{e}_2+\hat{e}_4^2}}{\sqrt{\kappa m_0}\sqrt{\hat{e}_3}},\\
v_4 &= \mp\sqrt{\frac{10}{3}}\frac{\hat{e}_4\sqrt{\hat{e}_3}}{\sqrt{\kappa m_0}\sqrt{-2\hat{e}_1\hat{e}_2+\hat{e}_4^2}},
\end{aligned}
$$

where we have defined shifted fluxes invariant under the shifts of t_a[20]

$$
\hat{e}_i \equiv e_i + \frac{\kappa_{ijk}m_jm_k}{2m_0}. \tag{3.67}
$$

For this solution to have a geometrical interpretation, we have to demand that sign $[m_0(-2\hat{e}_1\hat{e}_2+\hat{e}_4^2)] = \text{sign}\,[\hat{e}_3]$, $v_3 > 0$ and $(2\hat{e}_1\hat{e}_2 - \hat{e}_4^2) > 0$. Comparing this with the solution found in (3.15) we see that the additional constraint sign $[m_0 e_3] < 0$ is required for this solution to be supersymmetric.

To look at one explicit supersymmetric large volume and small string coupling example, we use the flux quantization condition (3.1) to express the results in terms of flux integers. Taking the limit $f_1 = f_2 = f_3 = f_4 =: f \gg 1$ leads to $v_1 = v_2 = 2v_3 = -v_4 \sim \frac{72}{\kappa^{1/3}}\frac{\alpha'}{\sqrt{|f_0|}}\sqrt{f}$. Therefore, for the internal volume, the string coupling and the potential we get

$$
vol_{(6)} = \kappa v_3(v_1v_2 - \frac{1}{2}{v_4}^2) \sim 9\times 10^4 \frac{(\alpha')^3}{|f_0|^{3/2}}\, f^{3/2}, \tag{3.68}
$$

$$
g_s = e^{\hat{\phi}} \sim 4\left|\frac{h}{f_0^{1/4}}\right| f^{-3/4}, \tag{3.69}
$$

$$
V_{\text{eff}} = -\sqrt{\frac{3}{10}}\,\frac{243}{1600\pi^8}\sqrt{\left|\frac{f_0^5}{f^9}\right|}\frac{h^4}{(\alpha')^4} \sim -9\times 10^{-6}\sqrt{|f_0^5|}\frac{h^4}{(\alpha')^4}\, f^{-9/2}. \tag{3.70}
$$

Gauge redundancies and counting of solutions. An interesting question is to ask how many physically different solutions there are for different values of the Kähler axions $b_a = \frac{m_a}{m_0}$. There are certain modular transformations of infinite order that act as shifts on the axions and relate equivalent vacua [22]. A integer shift of the Kähler axions

$$
b_a \to b_a + u_a, \quad u_a \in \mathbb{Z}, \forall a, \tag{3.71}
$$

corresponds to a shift of the $\hat{F}_2$ flux $m_a \to m_a + u_a m_0$. Now, since $|m_0|$ is (almost) fixed by tadpole cancelation, we see that physically inequivalent choices for m_a (and thus b_a) are defined modulo $|m_0|$. Consequently, once m_0 is fixed there are at most two different inequivalent solutions for different values of the b_a.

[20]Remember that there is a modular transformation that shifts the axions b_a by one.

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sector:	untwisted	Θ, Θ^3-twisted	Θ^2-twisted	$\sum$
fixed points/type:	—	16 $\mathbb{Z}_4$	12 $\mathbb{Z}_2$+ 4 $\mathbb{Z}_4$ ($\mathbb{Z}_2$)	—
complex structure:	1	—	6+0	1+6
Kähler:	$5 \to 4$(odd)	$16 \to 12$	$6+4 \to 5+4$	$5+26 \to 4+21$

Table 6: List of moduli before and after orientifold projection.

Validity of approximations. In order for the low energy supergravity approximation (leading order in α') to be valid we have to make sure that the dimensionless expansion parameter

$$\frac{\alpha'}{R^2} \sim f^{-1/2} \ll 1. \tag{3.72}$$

Moreover, we also want the string coupling to be small enough to be in a perturbative regime where we can safely neglect quantum (string loop) corrections. As we have observed above, $g_s \sim f^{-3/4}$. Therefore, by choosing $f \gg 1$ sufficiently large, we can ensure both conditions simultaneously.

Another important issue is the backreaction of the fluxes on the geometry: Namely, in the presence of background fluxes, the internal space is strictly speaking no longer a Calabi-Yau orientifold. However, we want to make sure that the low energy spectrum we assumed is still correct. For this to be true we must check that the mass scale of the (canonically normalized) Kähler moduli is sufficiently small compared to the mass scale of the massive Kaluza-Klein modes ($m_{\mathrm{KK}} \sim \frac{1}{R}$) which we neglected. Performing the calculations in the 4D Einstein frame, we find

$$m_{\tilde{v}_a} \sim f^{-9/4} \ll m_{\mathrm{KK}} \sim f^{-1/4}, \tag{3.73}$$

where $\tilde{v}_a := \frac{\delta v_a}{\kappa_{10} < v_a >}$ is normalized to give a canonical kinetic term in the Lagrangian. Clearly, their masses will be much smaller than the Kaluza-Klein masses if we choose $f \gg 1$ large.

4. Moduli stabilization in the twisted sectors

Fixed point structure and exceptional divisors. After having described the moduli stabilization in the untwisted sector, it remains to investigate the stabilization of the blow-up modes in the twisted sectors. Therefore let us briefly summarize the fixed point structure of our orientifold model (table 6). The exceptional divisor can be determined as follows: We start by modding out the $T^6 = T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)}$ by the $\mathbb{Z}_2$-action Θ^2. This yields 16 singularities of type $\mathbb{C}^2/\mathbb{Z}_2 \times T^2_{(3)}$, whose blow-up is given by 16 $\mathbb{C}P^1 \times T^2_{(3)}$. In a second step, we mod out this blown-up space $\widetilde{T^6/\mathbb{Z}_2}$ by the $\mathbb{Z}_2$-action Θ. The $\mathbb{C}P^1$s located at $\mathbb{Z}_2$ fixed points of the first two tori (cf. figure 2) get mapped into each other by Θ. Moreover, the two $\mathbb{Z}_2$ fixed points of the second torus are identified under the orientifold involution σ. This leaves us with $6 \to 5$ $\mathbb{C}P^1 \times T^2_{(3)}$ that contribute to the twisted Kähler moduli. Furthermore, the 6 $\mathbb{C}P^1$s at the $\mathbb{Z}_2$ fixed points can be tensored with the two 1-cycles on

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divisor	intersection type	intersection number
$T = \mathbb{C}P^1 \times T^2_{(3)}$	$T \circ T \circ T$	0
$T = \mathbb{C}P^1 \times T^2_{(3)}$	$T \circ T \circ \left[U = T^2_{(1)} \times T^2_{(2)}\right]$	$\beta = -2$
$T' = \mathbb{C}P^1 \times \mathbb{C}P^1$	$T' \circ T' \circ T'$	$\alpha = 8$

Table 7: List of intersection numbers.

the third torus to yield 12 twisted 3-cycles of topology $S^2 \times S^1$ (which contribute 6 twisted complex structure moduli). The 4 $\mathbb{C}P^1$s sitting at the $\mathbb{Z}_4$ fixed points of the first two tori remain invariant under this action and contribute 4 Kähler moduli (the sizes of the $\mathbb{C}P^1$s) to the twisted sectors. The 16 fixed loci of the Θ-action are $\mathbb{C}P^1 \times \{\text{point}\}$, where $\{\text{point}\}$ denotes one of the fixed points of the third torus (cf. figure 2). Two of these get identified by σ. Blowing-up results in $16 \to 12$ $\mathbb{C}P^1 \times \mathbb{C}P^1$, which give us the 12 Kähler moduli from the Θ^1, Θ^3 sectors.

Intersection numbers. In order to solve the F-term conditions for the twisted Kähler moduli, we need to calculate the various triple intersection numbers of the blow-up cycles. The results are listed in table 7. These results can be used to extend the F-term equations discussed above to include the twisted moduli.

It is important to note that there must be a hierarchy between the untwisted and twisted Kähler moduli,

$$|m_0| \ll |e_A| \ll |e_a|, \tag{4.1}$$

in order to remain within the Kähler cone [22]. This is the reason why, although there are non-vanishing intersection numbers linking the twisted sectors to the untwisted sector, the values at which the untwisted Kähler moduli are stabilized will not significantly change compared to the analysis of only the untwisted sector above.

Solutions to Kähler structure equations. For the b_a we have the same solutions as above $b_a = \frac{m_a}{m_0}$ where a now runs from 0 to 26.

For the v_a we have to solve the equations (3.63). The solution is

$$\begin{aligned}
v_1 &= \pm\sqrt{\frac{10}{3}}\frac{\hat{e}_2\sqrt{\hat{e}_3}}{\sqrt{\kappa m_0}\sqrt{(-2\hat{e}_1\hat{e}_2+\hat{e}_4^2)-\frac{\kappa}{\beta}(\hat{e}_5^2+\cdots+\hat{e}_{14}^2)}}, \qquad (4.2)\\
v_2 &= \pm\sqrt{\frac{10}{3}}\frac{\hat{e}_1\sqrt{\hat{e}_3}}{\sqrt{\kappa m_0}\sqrt{(-2\hat{e}_1\hat{e}_2+\hat{e}_4^2)-\frac{\kappa}{\beta}(\hat{e}_5^2+\cdots+\hat{e}_{14}^2)}},\\
v_3 &= \mp\sqrt{\frac{5}{6}}\frac{\sqrt{(-2\hat{e}_1\hat{e}_2+\hat{e}_4^2)-\frac{\kappa}{\beta}(\hat{e}_5^2+\cdots+\hat{e}_{14}^2)}}{\sqrt{\kappa m_0}\sqrt{\hat{e}_3}},\\
v_4 &= \mp\sqrt{\frac{10}{3}}\frac{\hat{e}_4\sqrt{\hat{e}_3}}{\sqrt{\kappa m_0}\sqrt{(-2\hat{e}_1\hat{e}_2+\hat{e}_4^2)-\frac{\kappa}{\beta}(\hat{e}_5^2+\cdots+\hat{e}_{14}^2)}},\\
v_5 &= \pm\sqrt{\frac{10}{3}}\frac{\hat{e}_5\sqrt{\kappa\hat{e}_3}}{\sqrt{m_0}\,\beta\,\sqrt{(-2\hat{e}_1\hat{e}_2+\hat{e}_4^2)-\frac{\kappa}{\beta}(\hat{e}_5^2+\cdots+\hat{e}_{14}^2)}},
\end{aligned}$$

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$$
\begin{aligned}
&\vdots\\
v_{14} &= \pm\sqrt{\frac{10}{3}}\frac{\hat{e}_{14}\sqrt{\kappa\hat{e}_3}}{\sqrt{m_0}\,\beta\,\sqrt{(-2\hat{e}_1\hat{e}_2+\hat{e}_4^2)-\frac{\kappa}{\beta}(\hat{e}_5^2+\cdots+\hat{e}_{14}^2)}},\\
v_{15} &= \pm\sqrt{\frac{10}{3}}\sqrt{-\frac{\hat{e}_{15}}{\alpha m_0}},\\
&\vdots\\
v_{26} &= \pm\sqrt{\frac{10}{3}}\sqrt{-\frac{\hat{e}_{26}}{\alpha m_0}}.
\end{aligned}
$$

As before, there are some additional conditions on the relative signs of the fluxes. To ensure reality of the Kähler moduli, we need to have

$$
\mathrm{sign}\left[\left(\frac{\hat{e}_3}{m_0((-2\hat{e}_1\hat{e}_2+\hat{e}_4^2)-\frac{\kappa}{\beta}(\hat{e}_5^2+\cdots+\hat{e}_{14}^2))}\right)\right] > 0, \tag{4.3}
$$

$$
\mathrm{sign}\left[\left(\frac{\hat{e}_A}{\alpha m_0}\right)\right] < 0, \quad \forall A = 15,\ldots,26. \tag{4.4}
$$

The volume and the string coupling constant are

$$
\begin{aligned}
vol_{(6)} &= \frac{1}{6}\kappa_{abc}v_a v_b v_c \qquad &(4.5)\\
&= v_3(\kappa v_1 v_2 - \frac{\kappa}{2}{v_4}^2 + \frac{\beta}{2}\sum_{A=5}^{14} v_A^2) + \frac{\alpha}{6}\sum_{A=15}^{26} v_A^3 \qquad &(4.6)\\
&= \frac{5}{3}\sqrt{\frac{5}{6}}\sqrt{\left|\frac{\hat{e}_3((2\hat{e}_1\hat{e}_2-\hat{e}_4^2)+\frac{\kappa}{\beta}(\hat{e}_5^2+\cdots+\hat{e}_{14}^2))}{\kappa m_0^3}\right|} + \frac{\alpha}{6}\sum_{A=15}^{26}\left(-\frac{10\hat{e}_A}{3\alpha m_0}\right)^{3/2}, \qquad &(4.7)\\
g_s = e^{\hat{\phi}} &= -\frac{5}{\sqrt{2}}\frac{p}{m_0}\frac{1}{\sqrt{vol_{(6)}}}. \qquad &(4.8)
\end{aligned}
$$

Due to the hierachy of fluxes mentioned above, the results for the untwisted sector do not deviate substantially from those obtained without taking the twisted sector into account.

Twisted complex structure moduli. As we saw above, including the twisted sector we now have 7 complex structure moduli to stabilize. The holomorphic 3-form is $\Omega(\tilde{z}) = Z^K(\tilde{z})a_K - \mathcal{F}_K(\tilde{z})b^K, K = 0,\ldots,6$. Equation (3.64) is still valid if we fix the normalization of Ω such that $\mathrm{i}\int\Omega\wedge\bar{\Omega} = 1$. For the twisted complex structure the $p_K, K = 2,\ldots,6$ are not constrained by the tadpole conditions. We can for example choose them to be $p_K = 0, K = 2,\ldots,6$ which would fix the corresponding complex structures $\mathrm{Im}(\mathcal{F}_K) = 0$. If we choose any of the $p_K, K = 2,\ldots,6$ to be non zero, the corresponding complex structure is fixed as

$$
\mathrm{Im}(\mathcal{F}_K) = -\frac{p_K}{4\sqrt{2}\,p}. \tag{4.9}
$$

The axions as derived above in (3.54) satisfy

$$
\sum_{h^{(2,1)}=0}^{7} p_i\xi^i = e_0 + \frac{e_a m_a}{m_0} + \frac{\kappa_{abc}m_a m_b m_c}{3m_0^2}, \tag{4.10}
$$

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where we have used $b_a = \frac{m_a}{m_0}$ and a, b, c run from 1 to 26.

5. Conclusions and Outlook

In this note we have worked out the moduli stabilization for a specific type IIA orientifold model, namely an orientifolded $T^6/\mathbb{Z}_4$ orbifold. The hope is that it will now be possible to add certain ingredients (D6-branes) in order to build a (semi-)realistic model which combines an MSSM(-like) particle content with realistic cosmological features, e.g., $\Lambda > 0$, without introducing new, unfixed moduli. This will be addressed in a forthcoming paper [36]. A summary of what needs to be done is outlined in the following. We would like to lift the stable AdS vacua derived above to meta-stable dS vacua in a controlled way. There has been a renewed interest in recent literature in investigating the possibility of D-term induced spontaneous supersymmetry breaking [40 – 43]. In analogy to the type IIB case, where U(1) gauge field fluxes on $D7$-branes (magnetized $D7$-branes) wrapping 4-cycles in the internal space were proposed as a means to generate D-terms (and F-terms) which spontaneously break $N = 1$ supersymmetry [37, 38], we propose to use gauge field fluxes on $D6$-branes to induce similar terms in the type IIA setup [39, 40]. However, no concrete, viable stringy realization of a D-term uplift to a meta-stable dS vacuum has been found so far. According to [41], a necessary prerequisite for constructing D-term contributions fully consistent with supergravity constraints is the existence of unfixed axions that can participate in a supersymmetric Higgs mechanism (Stückelberg mechanism) to form a massive U(1) vector. As we have seen above, such unfixed (complex structure) axions exist in our model. Therefore, it would be interesting to see if we can consistently apply D-term supersymmetry breaking in this class of models.

Moreover, we would like to incorporate stacks of intersecting $D6$-branes [44] so as to build a (semi-)realistic particle spectrum featuring standard model or MSSM (-like) gauge groups. It was demonstrated in [29] that the $T^6/\mathbb{Z}_4$ orientifold model under consideration can give rise to interesting particle phenomenology, such as a 3-generation Pati-Salam model, utilizing supersymmetric configurations of fractional $D6$-branes. However, since we are working in the framework of massive type IIA theory, the presence of $D8$-branes renders $D6$-brane configurations that preserve some supersymmetry much less generic (cf. [45]). Therefore, a careful investigation of all the constraints is crucial to fully understand the phenomenological viability of such models. This interesting topic will be the subject of future study.

Acknowledgments

It is a pleasure to thank Aaron Bergman, Jacques Distler, Raphael Flauger, Sonia Paban and Uday Varadarajan for many helpful comments, discussions and encouragement. The research of the authors is based upon work supported by the National Science Foundation under Grant Nos. PHY-0071512 and PHY-0455649, and with grant support from the US Navy, Office of Naval Research, Grant Nos. N00014-03-1-0639 and N00014-04-1-0336, Quantum Optics Initiative.

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